

Quantifying the Benefit of Wavelength-Add/Drop in WDM Rings with Distance-Independent and Dependent Traffic

Jane M. Simmons, *Member, IEEE, Member, OSA*, Evan L. Goldstein, *Member, IEEE, Member, OSA*, and Adel A. M. Saleh, *Fellow, IEEE, Member, OSA*

Abstract— One drawback to deploying WDM SONET rings is the potentially large amount of equipment necessary for their deployment. Wavelength-add/drop multiplexers potentially reduce the amount of required SONET terminal equipment by allowing individual wavelengths to optically bypass a node rather than being electronically terminated. We have quantified the maximum terminal-equipment savings attainable using wavelength-add/drop for rings carrying uniform traffic and rings carrying distance-dependent traffic. The analysis makes use of both an enumerative methodology, and a “super-node” approximation technique that is applicable to arbitrary ring size and inter-node demand. In both the uniform and distance-dependent traffic scenarios, maximum terminal-equipment savings are shown to rapidly increase, over the region of interest, with both network size and inter-node demand. The value of wavelength-add/drop is accordingly expected to grow rapidly in rings interconnecting numerous high-capacity nodes.

Index Terms— Add/drop multiplexers, ring bundling, SONET WDM rings, wavelength-add/drop

I. INTRODUCTION

THERE is a great deal of interest in designing networks composed of Wavelength Division Multiplexed (WDM) Synchronous Optical Network (SONET) rings, due to their large capacity and rapid restoration capabilities. The inherent redundancy of rings, combined with their simple topology, allow for restoration to occur on the order of tens of milliseconds. Several SONET ring configurations and their relative merits in terms of cost, capacity, and restoration capabilities are discussed in [1], [2]. Coupling WDM technology with SONET rings greatly increases capacity, thereby reducing the amount of required fiber and allowing for more graceful upgrades. WDM rings, and various protection options for such rings, are discussed in [2]-[4]; [5], [6] specifically examine WDM SONET rings.

One drawback to WDM SONET rings is the potentially large amount of equipment necessary for their deployment. In typical point-to-point WDM deployments, every wavelength is electronically terminated at every network node, regardless of whether that wavelength carries any traffic that is sourced or sunk at that node. Terminating every wavelength at every

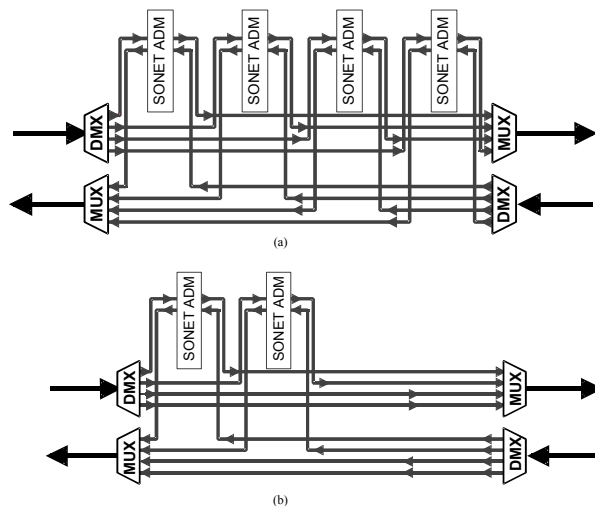


Fig. 1. a) A WDM SONET ring node in which all four wavelengths on a bi-directional pair of fibers are dropped at SONET ADMs. b) A WDM SONET ring node in which only two of the four wavelengths are dropped at SONET ADMs, thereby eliminating two of the ADMs.

node results in a significant amount of required SONET terminal equipment.

In order to relieve some of the terminal-equipment burden, one can take advantage of optical networking, where wavelengths may *optically bypass* a network node [6, 7]. For example, a wavelength add/drop multiplexer (WADM) can be deployed to *selectively* drop wavelengths at a node. (An overview of WADM technology is provided in [5], [6].) Thus, if a wavelength does not carry any traffic from, or for, a particular node, the WADM allows that wavelength to optically bypass the node, thereby eliminating the associated SONET terminal equipment.

To better illustrate the benefits of wavelength add/drop, consider the nodal equipment shown in Figs. 1a and 1b. In both figures, four wavelengths are multiplexed on a pair of bi-directional fibers. Fig. 1a illustrates a node on a typical WDM point-to-point SONET ring, where each of the four wavelengths is dropped at a SONET add/drop multiplexer (ADM). The SONET ADM demultiplexes the signal into lower-rate signals, allowing them to be dropped at the node, if necessary. If no lower rate signals need to be added or dropped, then an ADM is not required in the node. This scenario is illustrated in Fig. 1b. Here, the WADM drops only

two of the wavelengths, while the other two are passed through directly from the demultiplexer to the multiplexer, thereby eliminating two of the ADMs. ADMs that operate on OC-48 rate signals, for example, cost on the order of hundreds of thousands of dollars; thus, eliminating ADMs potentially represents a significant cost saving.

Wavelength add/drop will grow in importance as network traffic demands continue to dramatically increase. First, as traffic levels increase, it becomes more likely that entire wavelengths worth of traffic can be expressed through a node, thereby affording greater opportunity to implement optical bypass. This is borne out by our analysis below. Second, network operators are deploying WDM systems with increasingly more wavelengths multiplexed per fiber. For example, in a 100-wavelength WDM system, if wavelength add/drop is not implemented, then all 100 wavelengths would need to be terminated in SONET ADMs at a node even if only one of the wavelengths carries traffic that is sourced/sunk at that node. Thus, it is economically imperative that operators implement wavelength-add/drop.

We use the *through-to-total ratio*, as defined in [8], as a measure of the benefit of wavelength add/drop. We define *through-traffic* at a node as the traffic that ‘expresses through’ the node without being terminated in an ADM. *Total-traffic* is the traffic that either passes through or is terminated at the node. The through-to-total ratio is then simply the ratio of the through-traffic to the total-traffic, and represents the fractional terminal-equipment savings. In Fig. 1b, the through-to-total ratio, and thus the benefit provided by wavelength add/drop, is 2/4 or 50%, assuming that all wavelengths are fully packed.

This paper presents bounds on terminal-equipment savings for a ring supporting either uniform traffic or distance-dependent traffic between each pair of nodes. Maximum terminal-equipment savings are shown to rapidly increase, over the region of interest (SONET does not permit more than 16 nodes per ring), with both network size and inter-node demand. Also, uniform traffic yields greater savings than distance-dependent traffic, for the same amount of traffic.

Previous work (e.g., [6], [9]-[11]) has looked at virtual topology design and wavelength-assignment algorithms where the goal was minimizing properties such as the number of required wavelengths or the maximum propagation delay. In [12], a wavelength assignment algorithm is presented for rings carrying uniform all-to-all traffic that allows the minimum number of wavelengths to be used, even as additional nodes are added to the ring. Here, our focus is on minimizing the number of ADMs through the use of optical bypass, assuming the minimum number of wavelengths is used. Optical bypass in rings with uniform all-to-all traffic was presented in brief form in [13]. A similar problem was analyzed in [6] but only for the case of full-wavelength inter-node demand. The analysis here includes both full-wavelength and sub-wavelength demands. To the best of our knowledge, optical bypass with distance-dependent traffic has not been analyzed previously. Optical bypass in access rings, where the traffic is typically directed to a small number of hubs, was previously

analyzed in [6], [14], and is not included here.

In the next section, packing traffic into wavelengths is discussed. In Section III, the bounds for the maximal ADM savings are derived for rings carrying uniform traffic; the technique used to derive the bounds also enables one to produce constructions that achieve these optima. Section IV presents the ‘super-node’ approximation technique of [8] that can be used to extend the results of Section III to arbitrary ring-size and inter-node traffic. In Section V, the super-node approximation is used to derive bounds on the savings that can be achieved in rings with distance-dependent traffic, and these bounds are compared to those for uniform traffic.

II. WAVELENGTH PACKING

The analysis below assumes that each wavelength of the WDM ring supports a 4-fiber bi-directional SONET ring. This enables us to fully pack two working fibers (one clockwise, one counterclockwise), holding two empty ones in reserve for protection. ADMs are assigned on a per-wavelength basis (i.e., one ADM can terminate all four fibers). It is straightforward to extend the results to a 2-fiber ring, where half of the SONET TDM capacity is held in reserve for protection. It is also assumed that traffic is always routed over the shortest path, and that the minimum number of wavelengths possible is used.

Two traffic scenarios are considered in this paper: in Section III, we analyze uniform traffic, where one unit of traffic is sent from each node on the ring to every other node; in Section V, we examine distance-dependent traffic, where the number of traffic units sent between nodes increases as the inter-node distance decreases. For both scenarios, we first analyze the relatively simple case where the traffic unit is a full wavelength. Next, we consider more moderate demand where the traffic unit is only a fraction, $1/G$, of the bit rate transported by a wavelength, with $G > 1$. Thus, greater G indicates finer traffic granularity. Conceptually, we consider a wavelength to be composed of G channels, each carrying a fraction $1/G$ of a wavelength’s traffic capacity.

Each unit of traffic must be assigned to a particular channel, and the channels must be grouped together to form full wavelengths, where the traffic-assignment and bundling scheme that is used potentially affects the number of required ADMs. An ADM is required at a node for a particular wavelength if any of the channels comprising that wavelength add or drop traffic at that node. As illustration, consider the bi-directional ring shown in Fig. 2. Assume that each of the five nodes in the figure sends $1/2$ of a wavelength worth of traffic to every other node. The traffic is packed into three $1/2$ -wavelength (bi-directional) channels as shown in the figure (other configurations are possible), two of which need to be bundled together to form a full wavelength; the other wavelength is only half filled. The three channels, with their respective add/drop positions, are:

Channel 1: Add/Drops at Nodes 1, 3, and 4

Channel 2: Add/Drops at Nodes 1, 2, 4, and 5

Channel 3: Add/Drops at Nodes 2, 3, and 5

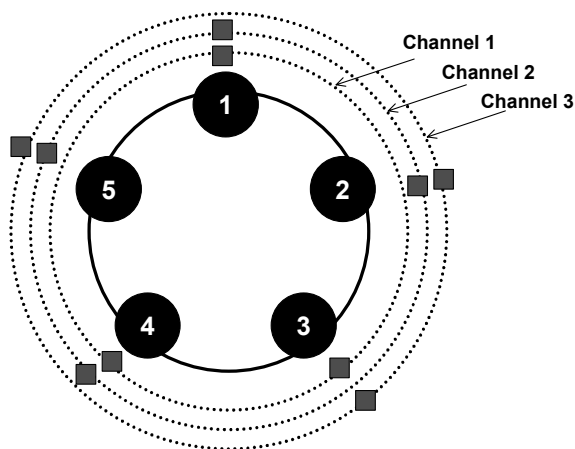


Fig. 2. Schematic illustration of a channel assignment for a five-node SONET ring with uniform all-to-all traffic. The channels add and drop traffic at locations marked by a square.

If Channels 1 and 3 are bundled together to form a wavelength, then ADMs are needed at all five nodes on this wavelength; the partially filled wavelength would be comprised of just Channel 2, and hence would need an ADM at Nodes 1, 2, 4, and 5. A total of 9 ADMs would be required. If, instead, Channels 1 and 2 are bundled together, then the full wavelength again requires ADMs at each of the five nodes, however, the remaining wavelength requires ADMs at only Nodes 2, 3, and 5. Thus, only 8 ADMs are required overall with this alternative bundling. In this small example, one ADM is saved due to judicious channel bundling. (Note that without optical bypass, 10 ADMs would be required, 5 for each wavelength.)

Throughout the paper it is assumed that a connection is carried by a single channel from source to destination; i.e., if a connection is placed in channel i of a particular wavelength at the source, then it remains on that channel until it is received by the destination node. Thus, the only ‘traffic grooming’ that we consider is the judicious assignment of connections to wavelength channels at the source node.

Alternatively, one can consider deploying switches at some, or all, of the nodes to *further* groom the traffic. The value of such grooming, in terms of further reducing the number of required ADMs, increases as the granularity of the inter-node traffic becomes finer. For full-wavelength inter-node traffic, no additional benefit is provided by the switches because at any point on the ring, a wavelength carries traffic that is destined for only one node. With sub-wavelength inter-node traffic, however, a given wavelength may carry traffic that is destined for several nodes. The switches can repack the wavelengths at each node such that a wavelength carries traffic for one, or a small number, of nodes; this may reduce the number of required ADMs, but typically at the expense of requiring more wavelengths. While the grooming function provided by the switch potentially results in fewer ADMs, it adds a great deal of complexity to the network management of the ring. We do not consider it further in this paper; however, it is likely that grooming through switches would produce

constructions that require somewhat fewer ADMs than we show below.

III. UNIFORM TRAFFIC

The first scenario we consider is uniform all-to-all traffic, where each node sends $1/G$ of a wavelength worth of traffic to every other node, for $G \geq 1$; the traffic is always routed over the shortest path. We focus on rings with N nodes, where N is odd, such that the shortest path is unambiguous. The results are similar for N even. For simplicity, below, we use the term ‘drop’ as opposed to add/drop.

It was shown in [5], [15] that, for N odd, the number of bi-directional channels needed to carry uniform all-to-all traffic is:

$$\frac{N^2 - 1}{8} \quad (1)$$

There are a total of $\frac{N(N-1)}{2}$ drops that must be assigned to the $\frac{N^2 - 1}{8}$ channels (i.e., one drop for each of the flows routed in each direction). Thus, the average number of drops per channel is:

$$\frac{N(N-1)}{2} \frac{8}{N^2 - 1} = \frac{4N}{N+1} \quad (2)$$

Equation (2) tends towards 4 as N increases.

A. Full-Wavelength Traffic

If each node transmits a full wavelength to each other node (i.e., $G=1$), then a channel represents a full wavelength; using Equation 1, there are $\frac{N^2 - 1}{8}$ required wavelengths. There

are $\frac{N(N-1)}{2}$ traffic flows in the clockwise direction and an equal number in the counterclockwise direction, with the traffic being symmetric in the two directions. One ADM is required for each source/destination pair, thus the minimum number of required (bi-directional) ADMs is:

$$\frac{N(N-1)}{2} \quad (3)$$

By contrast, in the absence of wavelength-add/drop (i.e., an ADM in every node for every wavelength), the number of required ADMs is:

$$\frac{N(N^2 - 1)}{8} \quad (4)$$

The through-to-total ratio, and hence the fractional ADM savings provided by wavelength-add/drop, is:

$$1 - \frac{\frac{N(N-1)}{2}}{\frac{N(N^2 - 1)}{8}} = \frac{N-3}{N+1} \quad (5)$$

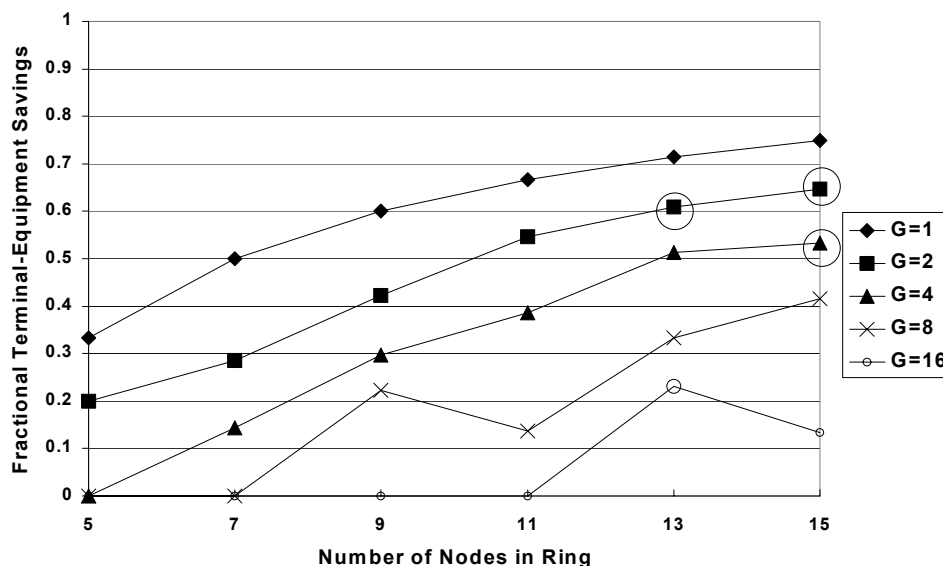


Fig. 3. Maximum fractional terminal-equipment savings achievable via wavelength-add/drop in a WDM SONET ring, for the uniform traffic scenario where each node sends $1/G$ of a wavelength worth of traffic to every other node. The parameter G is varied from 1 to 16. Terminal-equipment savings rise with node-to-node demand (i.e., smaller G) and ring size. Constructions corresponding to each point have been achieved except for the three encircled points.

As shown in Fig. 3, the resulting savings can be very significant, reaching a value of 75% in a 15-node ring.

B. Sub-Wavelength Traffic

For more moderate demand, where each node sends $1/G$ of a wavelength worth of traffic to every other node, and $G > 1$, G channels need to be bundled together to form full wavelengths. If $\frac{N^2 - 1}{8G}$ is not an integer then we assume that

$\left\lfloor \frac{N^2 - 1}{8G} \right\rfloor$ whole wavelengths are formed, with the remaining

channels forming a partial wavelength.

We use the notation $[i \ j \dots \ k]$ to indicate that a given channel drops at nodes $i, j \dots k$. For example, $[1 \ 5 \ 7]$ indicates that node 1 sends traffic to node 5, node 5 sends traffic to node 7, and 7 sends to 1. A direct path between any two given nodes appears in only one channel because we assume that there is one and only one $1/G$ of a wavelength sent between each pair of nodes. Thus, if a channel contains the path $[1 \ 4]$, that same path cannot appear in any other channel.

Statement: A channel must have a minimum of 3 drops

Proof: Assume there are only 2 drops, say nodes m and n , in the channel, such that traffic is sent from m to n , and from n to m . Such a channel cannot exist because one of these connections would be forced to cross more than $(N-1)/2$ links, and we assumed that traffic is always routed over the shortest path. However, a channel can have 3 drops: e.g., nodes 1, $(N+1)/2$, $(N+3)/2$.

1) $1/2$ -Wavelength Inter-node Demand

Suppose that each node sends one $1/2$ -wavelength worth of traffic to each other node, so that two channels must be combined to form a full wavelength.

Statement: Every combination of two channels with three drops each has a minimum of 5 drops in the union.

Proof: Assume channel 1 is: $[A \ B \ C]$. Channel 2 cannot contain the paths $[A \ B]$, $[B \ C]$, or $[C \ A]$ because each path can appear in only one channel. Also, channel 2 cannot be equal to $[A \ C \ E]$, for some E , because A and C are further than $(N-1)/2$ links apart (we know this because $[C \ A]$ is a path in channel 1). Nor can channel 2 equal $[A \ F \ B]$, for some F , because $[B \ A]$ is further than $(N-1)/2$ links apart. Thus, the two channels must differ in at least 2 drops, so that the union of the channels contains at least 5 drops.

Statement: Every combination of one channel with three drops and one with four drops has a minimum of 5 drops in the union.

Proof: Assume one channel has drops: $[A \ B \ C \ D]$. The channel with 3 drops can not be a subset of $[A \ B \ C \ D]$. If it were, it would contain at least one duplicate path e.g., the channel $[A \ C \ D]$ would contain a duplicate of the paths $[C \ D]$ and $[D \ A]$. Thus, the two channels must have at least one node that is not in common, e.g., $[A \ C \ E]$, yielding a union that contains at least 5 drops.

Using similar reasoning to above, we create Table 1 showing a partial listing of the minimum number of drops in the union of two channels.

From this table, we see that the minimum number of drops in a union of any two channels is 5. The most 'efficient' way of obtaining this is to take the union of a channel with 3 drops and a channel with 4 drops, thus accommodating 7 drops with only 5 ADMs (note, however, that not all such combinations yield a union of 5 drops). Thus, when dealing solely with full wavelengths, optimal ring bundlings arise by forming all wavelengths from such combinations, if possible.

As an example, for $N = 7$, one optimal bundling is:

$$\begin{aligned} \text{Wavelength \#1} &= \text{Union of } [1 \ 4 \ 5] \ \& \ [1 \ 2 \ 5 \ 7] \\ &= [1 \ 2 \ 4 \ 5 \ 7] \\ \text{Wavelength \#2} &= \text{Union of } [3 \ 6 \ 7] \ \& \ [1 \ 3 \ 5 \ 6] \\ &= [1 \ 3 \ 5 \ 6 \ 7] \\ \text{Wavelength \#3} &= \text{Union of } [2 \ 4 \ 6] \ \& \ [2 \ 3 \ 4 \ 7] \\ &= [2 \ 3 \ 4 \ 6 \ 7] \end{aligned}$$

(in general, optimal bundlings are not unique). This requires a total of 15 ADMs out of a possible $7 \bullet 3 = 21$ (i.e., ADMs at each of the 7 nodes on each of the 3 wavelengths). Wavelength-add/drop thus permits a terminal-equipment savings of 6/21. This is optimal, but it is less than the $\frac{N-3}{N+1} = \frac{1}{2}$ attainable with a full wavelength from each node to each other node, illustrating that finer granularity reduces the value of wavelength-add/drop.

As the number of nodes increases, it will eventually become impossible to form wavelengths solely from such combinations, since, as shown by Equation 2, the average number of drops per channel tends toward 4 with large N. Thus, it becomes necessary to form wavelengths with 6 drops; the most efficient way to do this is to combine a 4-drop channel and a 5-drop channel.

Consider an example with $N = 11$. Using Equation 1, there are a total of 15 channel, each comprising a half-wavelength's capacity; thus, 7 full wavelengths and one partially filled wavelength are formed. If the 7 full wavelengths are formed solely from combinations of 3-drop and 4-drop channels, then 6 drops will be left in the partial wavelength (the total number of drops is $\frac{N(N-1)}{2} = 55$). This yields a total of $7 \bullet 5 + 6 = 41$ ADMs. This bundling is in fact not optimal. In order to achieve an optimal bundling, it is necessary to form at least one full wavelength from a combination of a 4-drop channel and a 5-drop channel. One such optimal bundling is shown below:

$$\begin{aligned} \text{Wavelength \#1} &= \text{Union of } [1 \ 5 \ 6 \ 8 \ 10] \ \& \ [1 \ 6 \ 7 \ 8] \\ &= [1 \ 5 \ 6 \ 7 \ 8 \ 10] \\ \text{Wavelength \#2} &= \text{Union of } [2 \ 3 \ 5 \ 10] \ \& \ [5 \ 7 \ 10 \ 11] \\ &= [2 \ 3 \ 5 \ 7 \ 10 \ 11] \\ \text{Wavelength \#3} &= \text{Union of } [3 \ 6 \ 9 \ 10] \ \& \ [4 \ 6 \ 10] \\ &= [3 \ 4 \ 6 \ 9 \ 10] \\ \text{Wavelength \#4} &= \text{Union of } [4 \ 8 \ 9 \ 11] \ \& \ [3 \ 8 \ 11] \\ &= [3 \ 4 \ 8 \ 9 \ 11] \\ \text{Wavelength \#5} &= \text{Union of } [1 \ 4 \ 7 \ 11] \ \& \ [1 \ 2 \ 7] \\ &= [1 \ 2 \ 4 \ 7 \ 11] \\ \text{Wavelength \#6} &= \text{Union of } [1 \ 3 \ 7 \ 9] \ \& \ [3 \ 4 \ 9] \\ &= [1 \ 3 \ 4 \ 7 \ 9] \\ \text{Wavelength \#7} &= \text{Union of } [2 \ 4 \ 5 \ 8] \ \& \ [2 \ 5 \ 9] \\ &= [2 \ 4 \ 5 \ 8 \ 9] \\ \text{Wavelength \#8} &= [2 \ 6 \ 11] \end{aligned}$$

Table 1. Minimum Number of Drops in the Union of Two Channels

Drops in Channel 1	Drops in Channel 2	Minimum number of Drops in the union
3	3	5
3	4	5
3	5	6
3	6	6
4	4	6
4	5	6
4	6	7
5	5	7

This requires a total of 40 ADMs out of a possible $11 \bullet 8 = 88$ for a WDM SONET ring fully populated with ADMs. The resulting terminal-equipment savings of 55% is clearly substantial. However, again, this is smaller than $\frac{N-3}{N+1} = \frac{2}{3}$, the fractional savings achieved from full-wavelength inter-node traffic.

2. Finer Granularity

We can extend this approach to inter-node traffic smaller than $\frac{1}{2}$ wavelength, where the reasoning articulated above, through somewhat subtler combinatorics, yields the following results:

a) $\frac{1}{4}$ Wavelength Inter-node Traffic: any combination of four channels yields at least 7 drops, with the most efficient grouping being a union of three 4-drop channels and one 3-drop channel.

b) $\frac{1}{8}$ Wavelength Inter-node Traffic: any combination of eight channels yields at least 9 drops, with the most efficient grouping being a union of seven 4-drop channels and one 3-drop channel.

c) $\frac{1}{16}$ Wavelength Inter-node Traffic: for odd $N \leq 15$, it is only $N = 13$ and $N = 15$ that have at least 16 channels to form a full wavelength. In these cases, the number of ADMs is minimized when the full wavelength has an ADM at all N nodes.

C. Summary of Results

Using the above approach, one can obtain bounds on the fractional terminal-equipment savings obtainable from wavelength-add/drop. These are plotted in Fig. 3 for rings with between 5 and 15 nodes, roughly the range of practical interest. The savings are seen in general to rise steadily as both the size of the network and the inter-node demand increases. Wavelength-add/drop is seen to be particularly valuable in rings whose inter-node traffic exceeds about one-quarter of a wavelength. Indeed, the saturation of these curves suggests that at such high traffic levels, there is little incentive, from the point of view of equipment savings, to extend the SONET ring node limit beyond 16. However, for inter-node demand below one-quarter wavelength, only

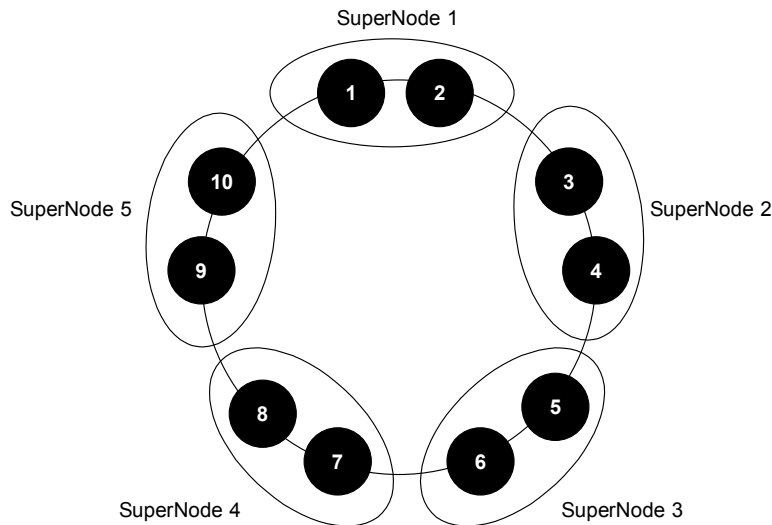


Fig. 4. The super-node approximation for 10 nodes. For a uniform inter-node demand of $\frac{1}{4}$ of a wavelength worth of traffic, the 10 nodes are partitioned into 5 super-nodes, each with 2 nodes. Each node within a super-node sends $\frac{1}{4}$ wavelength to each node in every other super-node, for a total of one full wavelength exchanged between each super-node pair.

modest savings are seen. Indeed, for inter-node demand of $\frac{1}{16}$ of a wavelength worth of traffic, no savings occur in rings with 11 or fewer nodes. To extract significant value from wavelength-add/drop in this low-traffic-volume regime would require an extension of the SONET ring standard beyond its current limit of 16.

Figure 3 in general suggests that as inter-node traffic volume increases, as is expected to occur swiftly in the public network, the value of wavelength-add/drop will increase accordingly. We should note, finally, that for all points in Fig. 3 excepting the three encircled ones, we were able to explicitly construct traffic bundlings exemplifying the computed optima. (There is nothing special about these three encircled points; manual construction becomes more difficult as N increases, because the number of possible combinations grows exponentially with N .)

IV. SUPER-NODE APPROXIMATION FOR UNIFORM TRAFFIC

For the case of full-wavelength traffic, the minimum number of required ADMs was easily seen to be precisely $\frac{N(N-1)}{2}$, one for each source/destination pair. Using the $\frac{N(N-1)}{2}$ bound, we can generate an approximation to the number of required ADMs for the scenario where $G > 1$. Using the technique of [8], we partition the N nodes of the ring into groups of \sqrt{G} nodes and form ‘super-nodes’ from each of these groupings, such that we have N/\sqrt{G} super-nodes, as illustrated in Fig. 4 for $N=10$ and $G=4$. Consider

sending $1/G$ of a wavelength worth of traffic from each of the \sqrt{G} nodes comprising one super-node to each of the \sqrt{G} nodes comprising any other super-node. In total, then, there are G connections from one super-node to the other, with the aggregate traffic being exactly a full wavelength. Treating each super-node as if it were a single node, we have N/\sqrt{G} nodes with a full wavelength of traffic sent from each node to every other node. Using the full-wavelength results of Section III.A, the through-to-total ratio for such an arrangement is:

$$\frac{\frac{N}{\sqrt{G}} - 3}{\frac{N}{\sqrt{G}} + 1} \tag{6}$$

and the minimum number of ADMs is:

$$\frac{\frac{N}{\sqrt{G}} \left(\frac{N}{\sqrt{G}} - 1 \right)}{2} \tag{7}$$

Each super-node is composed of \sqrt{G} real nodes, thus, each ADM required at a super-node translates into \sqrt{G} required ADMs at real nodes. Multiplying Equation (7) by \sqrt{G} yields the total number of required ADMs to be:

$$\frac{N \left(\frac{N}{\sqrt{G}} - 1 \right)}{2} \tag{8}$$

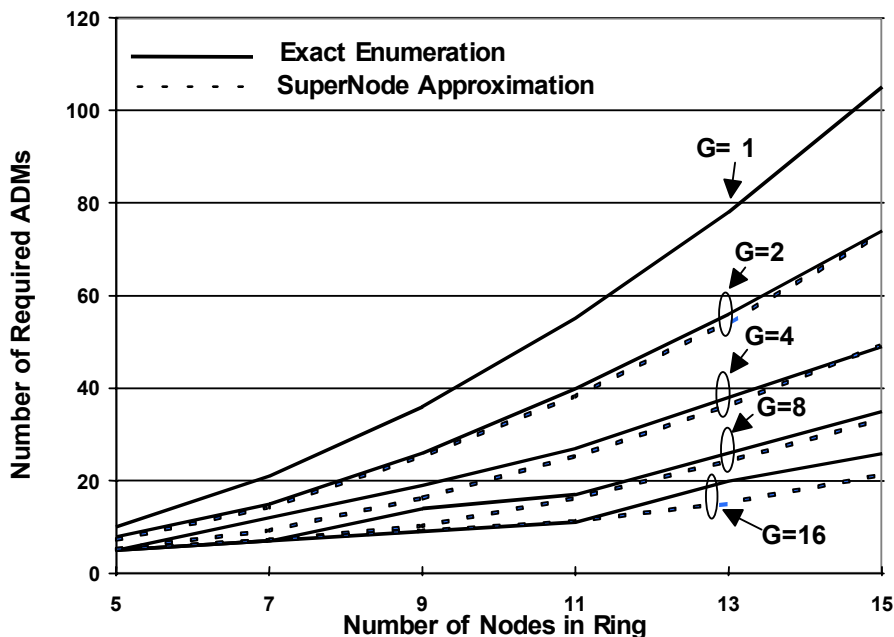


Fig. 5. Number of required ADMs in WDM SONET rings with uniform traffic. The solid lines indicate a lower bound on the number of required ADMs, whereas the dotted lines represent an approximation to the minimal required number, using the super-node approximation.

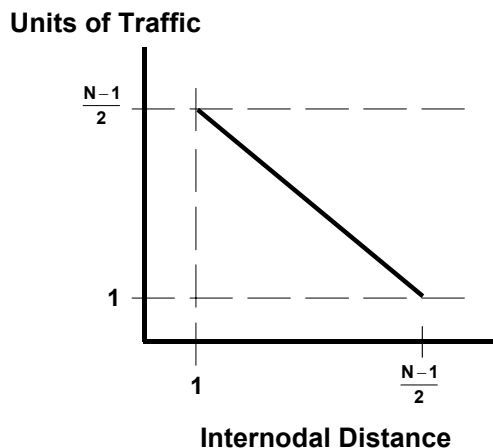


Fig. 6. Distance-dependent traffic demand pattern. The nodes furthest apart exchange one unit of traffic, and the inter-node traffic demand increases by one unit as the inter-node distance decreases by one link.

Although the quantities involved in the above equations (e.g., \sqrt{G}) do not always represent integers, we use Equation 8 to approximate the minimum number of ADMs required for arbitrary N and G . Note that the super-node construction does not account for traffic *between* the nodes comprising a super-node, thus we expect this approximation to underestimate the number of required ADMs. One ‘inefficient’ means of accounting for this missing traffic is to add an additional wavelength with ADMs at all N nodes. Thus, a strict upper-bound on the number of required ADMs is provided by adding a factor of N to Equation 8; this bound, however, becomes very loose as N increases. It is preferable to use the

approximation of Equation 8, which, as shown in Fig. 5, does an excellent job of predicting the number of required ADMs.

Figure 5 plots the number of required ADMs using both the bounding technique of Section III and the approximating super-node method. There is relatively good agreement between the curves, although, as expected, the super-node approximation underestimates the minimal required number of ADMs. In general, for a given node count N , the super-node approximation grows less accurate at low inter-node demand (large G). This is because a super-node then represents a number of actual nodes (\sqrt{G}), whose internal traffic is neglected. What is remarkable is the accuracy one nevertheless obtains from the super-node approximation over the plotted range. Should rings with yet larger number of nodes become important in the future, the exact combinatoric approach of Section III would likely prove unwieldy.

V. DISTANCE-DEPENDENT TRAFFIC

In real networks, traffic demand has historically been correlated with distance, with nodes that are closer in distance exchanging more traffic. In this section, we incorporate this feature using the distance-dependent traffic demand curve shown in Fig. 6, which is chosen because of its analytic simplicity and reasonable depiction of reality. The amount of traffic between the most distant nodes is one unit; the traffic demand increases by one unit as the inter-node distance decreases by one link. We consider bi-directional rings, so that the shorter path between two nodes is used as the internodal distance.

Consider the traffic carried in a given direction on any *link* in the ring. The traffic is composed of:

$$\begin{aligned}
1 \text{ Connection of Distance } 1 &\Rightarrow \frac{N-1}{2} \text{ Traffic Units} \\
2 \text{ Connections of Distance } 2 &\Rightarrow 2 \left[\frac{N-1}{2} - 1 \right] \text{ Traffic Units} \\
3 \text{ Connections of Distance } 3 &\Rightarrow 3 \left[\frac{N-1}{2} - 2 \right] \text{ Traffic Units} \\
&\vdots \\
\frac{N-1}{2} \text{ Connections of Distance } \frac{N-1}{2} &\Rightarrow \frac{N-1}{2} \text{ Traffic Units}
\end{aligned}$$

The number of traffic units on each link is then:

$$\sum_{i=1}^{(N-1)/2} i \left[\frac{N-1}{2} - (i-1) \right] = \frac{(N^2-1)(N+3)}{48} \quad (9)$$

Equation 9 represents the minimum number of channels (each channel carrying one traffic unit) that are required to carry the distance-dependent traffic of Fig. 6.

The number of units of traffic sent by each node, in each direction, is:

$$\sum_{i=1}^{(N-1)/2} i = \frac{N^2-1}{8} \quad (10)$$

A. Full-Wavelength Traffic

We first consider the scenario where the traffic unit is a full-wavelength ($G=1$); i.e., the nodes that are furthest apart exchange 1 wavelength of traffic; nodes that are immediately adjacent exchange $\frac{N-1}{2}$ wavelengths. Using Equation 10,

there are a total of $\frac{N(N^2-1)}{8}$ full-wavelength traffic flows in

the clockwise direction and an equal number in the counterclockwise direction, with the traffic being symmetric in the two directions. One ADM is required for each full wavelength source/destination pair, thus the minimum number of required (bi-directional) ADMs is:

$$\frac{N(N^2-1)}{8} \quad (11)$$

By contrast, in the absence of wavelength-add/drop, the number of required ADMs is (using Equation 9):

$$\frac{N(N^2-1)(N+3)}{48} \quad (12)$$

The through-to-total ratio, and hence the fractional ADM savings provided by wavelength-add/drop, is:

$$1 - \frac{\frac{N(N^2-1)}{8}}{\frac{N(N^2-1)(N+3)}{48}} = \frac{N-3}{N+3} \quad (13)$$

As with uniform traffic, the benefit of wavelength-add/drop with distance-dependent traffic increases with the size of the

ring, reaching a value of 67% in a 15-node ring.

B. Sub-Wavelength Traffic

For the case of sub-wavelength inter-node traffic, where $G > 1$, we again use the super-node technique to approximate the number of required ADMs. Assume that a super-node is comprised of X nodes. Consider the number of traffic units sent from Super-node A to Super-node B, where these represent two super-nodes that are at a maximum distance from each other. The nodes that are furthest apart in Super-nodes A and B are at the maximum inter-node distance for the ring, and thus exchange 1 unit of traffic. (For example, in Fig. 4, Super-nodes 1 and 3 are at the maximum inter-super-node distance, and Nodes 1 and 6 are at the maximum inter-node distance.) As the inter-node distance decreases by 1, the number of traffic units increases by 1. The traffic from Super-node A to Super-node B is then composed of:

$$\begin{aligned}
\text{Node 1 in Super-node A} &\rightarrow \text{Super-node B:} \\
&1 + 2 + \dots + X-1 \text{ Traffic Units} \\
\text{Node 2 in Super-node A} &\rightarrow \text{Super-node B:} \\
&2 + 3 + \dots + X \text{ Traffic Units} \\
&\vdots \\
\text{Node X in Super-node A} &\rightarrow \text{Super-node B:} \\
&X + (X+1) + \dots + (2X-1) \text{ Traffic Units}
\end{aligned}$$

Summing this traffic yields a total of X^3 traffic units sent from any super-node to the super-node most distant from it. As the super-nodes get closer, each of the X^2 connections between the super-nodes is X units closer, and thus generates an additional X traffic units. Thus, the inter-super-node traffic increases by a total of X^3 traffic units as the super-nodes grow closer. In Fig. 4, where X equals 2, the total amount of traffic sent from super-node 1 to 3 is 8 units (i.e., 2^3) and from super-node 1 to 2 is 16 units (i.e., $2^3 + 2^3$), where each unit is $1/G$ of a wavelength.

The super-node size should be chosen such that each super-node sends an integral number of wavelengths to every other super-node. Thus, for $1/G$ of a wavelength worth of inter-node traffic, the super-node size, X , is chosen to be $G^{1/3}$, which yields an inter-super-node traffic pattern that precisely mimics the solid line in Fig. 6 (with the horizontal-axis representing super-node distance). Treating each super-node as if it were a single node, we have $N/G^{1/3}$ nodes. Using the full-wavelength results of Equations 11 and 13, respectively, the number of required ADMs in such a configuration is:

$$\frac{N}{G^{1/3}} \left[\left(\frac{N}{G^{1/3}} \right)^2 - 1 \right] \quad (14)$$

and the fractional ADM savings is:

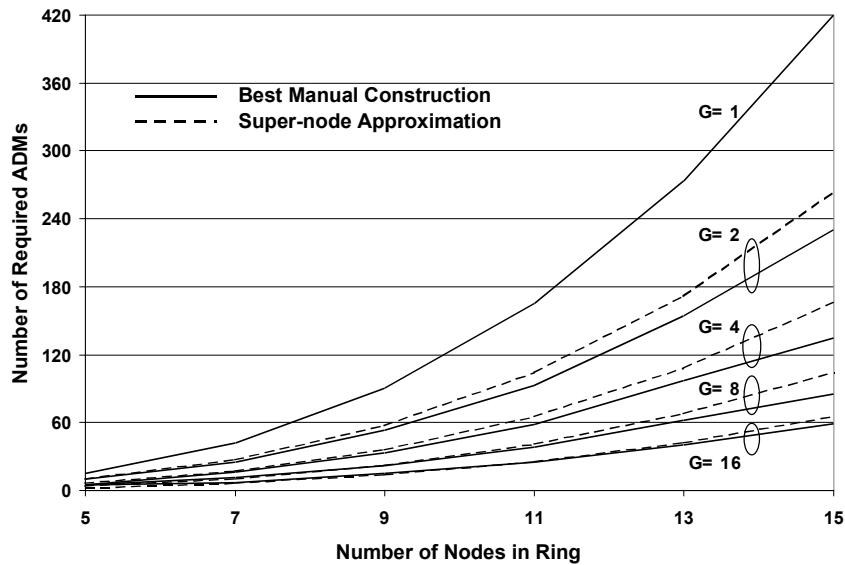


Fig. 7. Number of required ADMs in WDM SONET rings with distance-dependent traffic. The solid lines indicate the number of required ADMs in the best manual constructions. The dotted lines represent an approximation to the minimal required number, using the super-node approximation.

$$\frac{\frac{N}{G^{1/3}} - 3}{\frac{N}{G^{1/3}} + 3} \quad (15)$$

If we assume that every ADM required in a super-node corresponds to $G^{1/3}$ ADMs in real nodes, then, multiplying Equation 14 by $G^{1/3}$, the overall number of required ADMs is:

$$N \left[\frac{\left(\frac{N}{G^{1/3}} \right)^2 - 1}{8} \right] \quad (16)$$

In Fig. 7, we show the number of required ADMs as predicted by Equation 16 for various ring sizes and traffic demand granularities. We compare this approximation to the best results we were able to attain through manual constructions (the manual constructions are not necessarily optimal). The number of required ADMs predicted by Equation 16 is seen to be higher than the number of ADMs required in our constructions; the difference is larger when N is large and G is small (excepting $G=1$, which is exact). The overestimate occurs because fewer than all $G^{1/3}$ nodes in each super-node may need to drop a particular inter-super-node wavelength. For example, assume that G equals 8, such that the unit of traffic is $1/8$ of a wavelength and the super-node size is two. As the inter-super-node distance grows, a point is reached (assuming N is large enough) where one node sends 8 units of traffic (i.e., a full wavelength) to a node in another super-node. This particular wavelength only needs to drop in one node, not both nodes, of each super-node. As N increases and G decreases, there is a greater occurrence of inter-super-node wavelengths that need to be dropped at only a fraction of the nodes comprising the super-nodes. Thus, to better approximate the overall number of required ADMs, the

number of ‘super-node ADMs’, as given by Equation 14, should ideally be multiplied by a factor that is somewhat smaller than $G^{1/3}$ (as opposed to exactly $G^{1/3}$), where the factor depends on N and G . Nevertheless, for simplicity, we use Equation 16, which, as shown in Fig. 7, is a reasonable approximation to the number of required ADMs.

C. Uniform vs. Distance-Dependent Ring Traffic

In order to compare the savings that can be obtained using optical bypass with uniform traffic as opposed to distance-dependent traffic, we normalize the traffic such that there is an equal amount of traffic sourced at each node. For $1/G$ uniform inter-node traffic, the total amount of traffic sourced at each node in either the clockwise or counterclockwise direction is $\frac{N-1}{2G}$; for $1/G$ distance-dependent traffic, the

total in either direction is $\frac{N^2-1}{8G}$, as was shown in Equation

10. Thus, for comparison purposes, we use $1/G$ uniform traffic and $1/G'$ distance-dependent traffic, where G' equals $\frac{N+1}{4}G$.

The comparison between the two traffic demand patterns is shown in Fig. 8 for a range of ring sizes and for G equal to 1, 4, and 16. The savings with uniform traffic is greater than or equal to that of distance-dependent traffic in all cases. This is expected because distance-dependent traffic favors connections that are close, resulting in less through traffic. The differences are smaller for finer granularity traffic (i.e., larger G); more connections need to be bundled together to form a full wavelength so that it is less likely a wavelength can bypass a node in either traffic demand scenario.

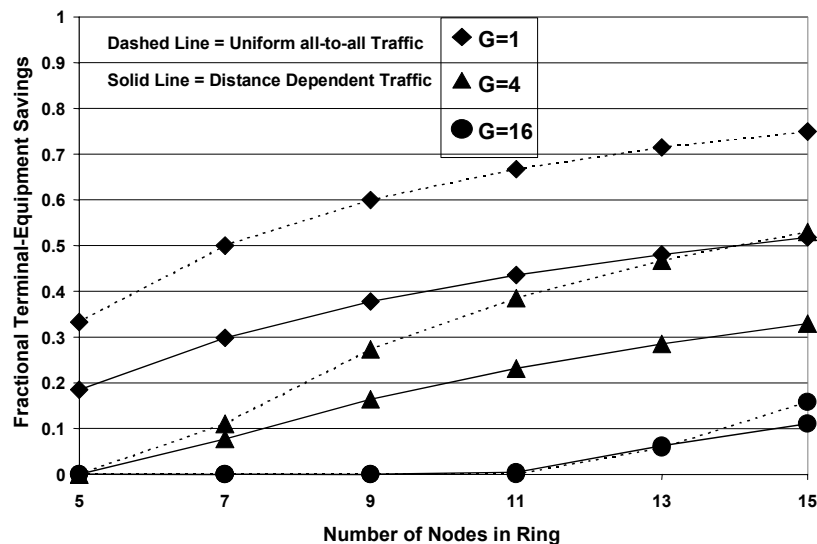


Fig. 8. Fractional terminal-equipment savings with uniform traffic vs. distance-dependent traffic on a ring. The inter-node demand for the uniform traffic scenario is $1/G$ of a wavelength, for $G = 1, 4, 16$. The inter-node demand for the distance dependent traffic is adjusted accordingly so that the amount of traffic sourced at each node equals that of the uniform traffic scenario.

VI. CONCLUSION

We have shown that significant benefit can be mined from implementing wavelength add/drop in WDM SONET rings. For the case of uniform traffic, we have devised a traffic-bundling methodology that enables one to both quantify the maximum terminal-equipment savings attainable using wavelength-add/drop, and to produce constructions that achieve these optima. In addition, we have applied the super-node technique to both the uniform traffic and distance-dependent traffic scenarios, so that the fractional ADM savings can be approximated for arbitrary ring size.

Maximum terminal-equipment savings were shown to increase with both network size and inter-node demand, indicating that the role of wavelength add/drop will become more important as networks continue their dramatic growth. Furthermore, the role of optical bypass is not limited to eliminating SONET terminating-equipment. For example, in an IP-over-WDM environment, the ability to optically bypass some IP routers potentially alleviates the processing bottleneck posed by current routing technology. Thus, while this analysis quantifies only equipment savings, these savings may permit substantial simplifications in packet-traffic routing.

The traffic scenarios considered in this paper were static and were idealized to enable analytic tractability. However, as configurable WADM technology matures, network operators will have the ability to dynamically rearrange the network configuration in response to arbitrary and changing traffic patterns. This latter subject is likely to emerge as an area of substantial practical importance.

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